Feeling sick on Friday, Rachael Willingham went to the doctor, who gave her an order for the new coronavirus test and sent her to a mobile clinic the Colorado health department had set up. When she arrived at 9:45 a.m., a half-dozen police officers were blocking the entrance. She returned to the clinic that afternoon, but was told testing was over for the day.

Ms. Willingham called a number for the state health department and was told to come back Saturday at noon. She did, only to find testing had been moved to the Denver Coliseum. When she got to the stadium, she waited in a line of hundreds of cars for almost two hours, only to be turned away again with no explanation.

“It’s making this worse.” the 29-year-old said of the confusion over testing. “It’s contributing
As cases of Covid-19 have exploded across the U.S., state and local governments are taking on the task of testing for the coronavirus that causes it—and they have been quickly overwhelmed. Slowed by equipment shortages and struggling to keep pace, officials have set up a chaotic patchwork of testing sites, with access varying wildly from one place to another. Now some states and counties are pulling back, using their limited resources to test only the most vulnerable.

Testing is critical for infected people to get the treatment they need and for health officials to accurately track the spread of the coronavirus. But it has proceeded far slower than experts say is necessary, in part due to a slow federal response.

That reality is playing out daily across the country, as Americans try but often fail to get help from their local governments.

The Colorado health department has moved the mobile testing clinic in Denver that Ms. Willingham attempted to access to the tiny mountain town of Telluride, six hours away, where the state's ski towns have been hard hit. Officials said they had no plans to reopen one in Denver and have pivoted to targeted testing of the most vulnerable individuals there. The state can process about 250 tests a day, but a backlog means results take as many as five days, a health department spokesperson said.

After initially offering widespread testing, Minnesota officials retrenched Tuesday, limiting testing to health-care workers, inpatients at hospitals and people in group living facilities.

Kris Ehresmann, commissioner of the Minnesota health department, said high demand made testing everyone who might be infected impossible. “Reserving these laboratory tests for the sickest individuals is the right thing to do,” she said.

On Wednesday, South Dakota Gov. Kristi Noem said its state lab was forced to temporarily stop processing tests because it ran out of the necessary supplies.

Ms. Noem said they have been trying for weeks to get more of the chemicals used to process the tests from suppliers and had asked Vice President Mike Pence to help.

When the coronavirus first hit the U.S., all tests were initially sent to the Centers for Disease Control and Prevention, which was slow to make them available. As cases surged earlier this month, the federal government began allowing state and local authorities, as well as private companies, to conduct the tests themselves, hoping to speed the process.
But local rollouts have been slowed by a cascade of problems. Many areas lack enough test kits. Some are short on protective gear that providers need to conduct the tests. And the volume of people trying to get tested has become a logistical nightmare.

Anxious for help as cases rise, states including Minnesota and New Mexico have called on the federal government and manufacturers for assistance obtaining supplies such as personal protective equipment and testing devices.

The Trump administration, meanwhile, has said it is shifting its strategy for containing the outbreak away from testing and toward social distancing.

The Department of Health and Human Services didn’t immediately respond to a request for comment. Federal health officials have said in congressional testimony that they are aware there are possible shortages of chemicals used in the tests. They have said they have ramped up the deployment of test kits to states and are supporting efforts for more drive-through tests.

In Ojai, Calif., north of Los Angeles, Colleen Byrum tried in vain last week to get a test after she began running a 103 degree fever. At first, the 61-year-old was told by a receptionist who picked up the phone at Ojai Valley Community Hospital to come to the emergency parking lot for a test,
she said. But when she arrived, she was told that wasn’t possible and she should see her doctor Monday.

Upset, Ms. Byrum called the emergency room back and was given a number for the CDC, which went to a recorded message telling her to call back Monday. After calling the ER again, she was given the number to the county infectious-disease response team, which also led to a voice message. The ER finally told her she didn’t meet the criteria to get tested because she hadn’t traveled to China, been on a cruise ship or been in contact with someone known to have the coronavirus infection. Finally, she was able to reach another local doctor, who said there weren’t enough testing kits to go around.

“It’s hard to advocate for yourself when you’re sick. It is not only physically exertional, it is emotionally exertional,” she said.

In some states, officials say capacity is improving and anyone who is sick should be able to get tested.

This week, New York City officials said their own testing capacity would be at 5,000 samples per day by Thursday, and Gov. Andrew Cuomo said the state would test 6,000 samples per day as well.

But even private labs, which states are turning to for help, are struggling to catch up.

FoundCare, a drive-through testing site in Palm Beach County, one of the hardest-hit areas in Florida, had been collecting samples for days when it abruptly stopped.

A FoundCare employee who answered the phone said the clinic was no longer accepting appointments because of the “high volume of calls and no supply” of the tests.

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